NLO Exclusive Evolution Kernels.

A.V. Belitsky[a], **A. Freund**[b], **D. Müller**[c]

[a] *C.N. Yang Institute for Theoretical Physics*
State University of New York at Stony Brook
NY 11794-3800, Stony Brook, USA

[b] *INFN, Sezione di Firenze, Largo E. Fermi 2*
50125, Firenze, Italy

[c] *Institut für Theoretische Physik, Universität Regensburg*
D-93040 Regensburg, Germany

Abstract

We outline a formalism used for a construction of two-loop flavor singlet exclusive evolution kernels in the $\overline{\mathrm{MS}}$ scheme. The approach is based on the known pattern of conformal symmetry breaking in $\overline{\mathrm{MS}}$ as well as constraints arising from the superconformal algebra of the $\mathcal{N} = 1$ super Yang-Mills theory.

Talk given at the
8th International Workshop on Deep Inelastic Scattering and QCD
Liverpool, April 25-30, 2000

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A. V. Belitsky[a], A. Freund[b], D. Müller[c]
[a] *C.N. Yang ITP, SUNY at Stony Brook*
NY 11794-3800, Stony Brook, USA

[b] *INFN, Sezione di Firenze, Largo E. Fermi 2*
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[c] *Institut für Theoretische Physik, Universität Regensburg*
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We outline a formalism used for a construction of two-loop flavor singlet exclusive evolution kernels in the $\overline{\mathrm{MS}}$ scheme. The approach is based on the known pattern of conformal symmetry breaking in $\overline{\mathrm{MS}}$ as well as constraints arising from the superconformal algebra of the $\mathcal{N}=1$ super Yang-Mills theory.

1 Q^2 evolution of SPD

Exclusive processes provide an indispensable information for a construction of a unique picture of hadron wave functions, Ψ. Its lowest Fock components (integrated over different transverse momentum configurations of partons) go under the name of distribution amplitudes, $\phi(x)$. Being a fundamental characteristic, Ψ defines all other inclusive and exclusive observables. A product of a wave function and a complex conjugate with fixed transverse momentum

$$\phi\left(x, \eta, \Delta_\perp\right) \sim \int d^2 k_\perp \Psi^* \left(\tfrac{x+\eta}{2}, k_\perp + \tfrac{\Delta_\perp}{2}\right) \Psi \left(\tfrac{x-\eta}{2}, k_\perp - \tfrac{\Delta_\perp}{2}\right), \tag{1}$$

define a correlation function called skewed parton distribution (SPD). It generalizes its predecessor, — conventional inclusive density well known from DIS, to non-zero values of skewedness η and $\Delta_\perp$. A peculiar feature of the SPDs is that they have a very different behaviour depending on the kinematical regime, i.e. an interplay of x and η. Depending on the difference in the momentum fractions between the left- and right-hand-side of the parton ladder the SPDs behave like a regular parton distribution or like a distribution amplitude. Particular Mellin moments w.r.t. momentum fraction x give hadron (and real Compton scattering) form factors and angular momenta of constituents.

In QCD the leading twist SPD is defined as a Fourier transform to the momentum fraction space of a light-ray operator constructed from φ-parton fields and sandwiched between hadronic states non-diagonal in momenta, schemati-

cally given by ($\Delta = p' - p$)

$$\phi(x,\eta,\Delta_\perp|Q) = \frac{1}{2\pi}\int dz_- e^{ixz_-} \langle h(p')|\ \varphi^\dagger(-z_-/2)\varphi(z_-/2)\big|_Q\,|h(p)\rangle \qquad (2)$$

The logarithmic Q-scale dependence of ϕ arises due to a light-like separation of partons and is governed by a renormalization group equation. The generalized skewed kinematics for corresponding perturbative evolution kernels can unambiguously be restored from the conventional exclusive one $\eta = 1$.

$$\frac{d}{d\ln Q^2}\boldsymbol{\phi}(x|Q) = \boldsymbol{V}\left(x,y|\alpha_s(Q)\right)\otimes\boldsymbol{\phi}(y|Q), \qquad (3)$$

where $\tau_1 \otimes \tau_2(x,y) \equiv \int_0^1 dz\,\tau_1(x,z)\tau_2(z,y)$ defines the exclusive convolution and $\boldsymbol{\phi} = \left(\phi^Q,\phi^G\right)$ is the vector of the quark and gluon distributions and $\boldsymbol{V}$ is a matrix of evolution kernels. Thanks to conformal invariance of classical QCD Lagrangian the leading order kernels having the structure $\boldsymbol{V}^{(0)}(x,y) = \theta(y-x)\boldsymbol{f}(x,y) \pm \theta(x-y)\boldsymbol{f}(\bar{x},\bar{y})$ can be diagonalized in the basis spanned by Gegenbauer polynomials $C_j^\nu(x)\otimes\boldsymbol{V}^{(0)}(x,y) = \boldsymbol{\gamma}_j^{(0)}C_j^\nu(y)$ with forward anomalous dimensions (ADs) $\boldsymbol{\gamma}_j^{(0)}$. Beyond this level conformal symmetry is violated by quantum corrections and a diagonal AD matrix $\boldsymbol{\gamma}_j$ gets promoted to a triangular one $\boldsymbol{\gamma}_{jk}$, $k \leq j$. Thus $\boldsymbol{V} = \boldsymbol{V}^{\mathrm{D}} + \boldsymbol{V}^{\mathrm{ND}}$ with $\boldsymbol{V}^{\mathrm{ND}} \propto \mathcal{O}(\alpha_s^2)$. An efficient formalisms to tackle the problem which eludes explicit multi-loop exercise and is based on the use of special conformal anomalies which produce the non-diagonal part, $k < j$, of $\boldsymbol{\gamma}_{jk}$, converted into exclusive kernels $\boldsymbol{V}^{\mathrm{ND}}$; and relations resulting from $\mathcal{N} = 1$ SUSY Ward identities which connect diagonal part of the kernels, $\boldsymbol{V}^{\mathrm{D}}$, and allows to reconstruct all channels from a given QQ sector deduced by explicit evaluation of two-loop graphs.

2 Using conformal symmetry

Conformal operators which are Gegenbauer moments of ϕ, $C_j^\nu(x)\otimes\phi(x) \sim \langle h'|\mathcal{O}_{jj}|h\rangle$, build an infinite dimensional irreps of the collinear conformal algebra $so(2,1)$. Conformal Ward identities derived for the Green function with conformal operator insertion $\mathcal{G} \equiv \langle \mathcal{O}_{jk}\prod_i \varphi_i\rangle$ in the regularized QCD allows, by means of algebra of dilatation $\mathcal{D}$ and special conformal transformation $\mathcal{K}$, to prove a matrix constraint for ADs $\boldsymbol{\gamma}$ and special conformal anomaly $\boldsymbol{\gamma}^c$

$$[\mathcal{D},\mathcal{K}_-]_- = i\mathcal{K}_- \qquad \Rightarrow \qquad \left[\boldsymbol{a} + \boldsymbol{\gamma}^c + 2\tfrac{\beta}{g}\,\boldsymbol{b},\boldsymbol{\gamma}\right]_- = 0, \qquad (4)$$

with α_s-independent matrices $\boldsymbol{a}$ and $\boldsymbol{b}$ and QCD beta function $\beta = \frac{\alpha_s}{4\pi}\beta_0 + \cdots$. The solution of the above equation with available one-loop conformal anomalies

$\boldsymbol{\gamma}^c$ implies the following form of the nondiagonal part of the NLO kernel

$$\boldsymbol{V}^{\mathrm{ND}(1)}(x,y) = -(\mathcal{I}-\mathcal{D}) \left\{ \dot{\boldsymbol{V}} \otimes \left(\boldsymbol{V}^{(0)} + \frac{\beta_0}{2}\,\mathbb{1} \right) + \left[\boldsymbol{g} \otimes, \boldsymbol{V}^{(0)} \right]_- \right\} (x,y), \quad (5)$$

where $(\mathcal{I}-\mathcal{D})$ projects out the diagonal part $\gamma^{(1)}_{jj}$. Here $\dot{\boldsymbol{V}}$ is given mostly by a logarithmic modification of LO kernels $\boldsymbol{f} \to \boldsymbol{f} \ln \frac{x}{y}$ plus an addendum, while $\boldsymbol{g}$ is a kernel whose conformal moments are proportional to a $\boldsymbol{w}$ part of $\boldsymbol{\gamma}^c = -\boldsymbol{b}\boldsymbol{\gamma}^{(0)} + \boldsymbol{w}$.

3 Using $\mathcal{N}=1$ SUSY

The last problem is to find $\boldsymbol{V}^{\mathrm{D}}$. Although it seems straightforward to solve, since the Gegenbauer moments $\boldsymbol{V}^{\mathrm{D}}_{jk} = \delta_{jk}\boldsymbol{\gamma}_j$ coincide with forward ADs calculated to NLO presently, practical inversion is extremely hard to handle. The main difficulty being kernels stemming from crossed-ladder type diagrams which we called $\boldsymbol{G}$-functions. Since the conformal symmetry breaking part has been previously fixed, we can assume conformal covariance for the ADs. If one puts (Majorana) quarks into adjoint representation of $SU(N_c)$, the classical "QCD" Lagrangian enjoys $\mathcal{N}=1$ SUSY. In perturbative calculations (with SUSY preserving regularization) this simply means the following identification of Casimir operators: $C_F = 2T_F = C_A \equiv N_c$. From the commutator of the dilatation and translational SUSY generators $[\mathcal{Q},\mathcal{D}]_- = \frac{i}{2}\mathcal{Q}$ applied to the Green functions $\mathcal{G}$ one finds six constraints for eight $\boldsymbol{G}$-functions for even and odd parity sectors. Since the ${}^{QQ}G$-function in the QQ channel is explicitly known the other ones can be unambiguously reconstructed[1] and colour factors trivially restored. The full NLO kernel has now the following form

$$\boldsymbol{V}^{(1)} = -\dot{\boldsymbol{V}} \otimes \left(\boldsymbol{V}^{(0)} + \frac{\beta_0}{2}\,\mathbb{1} \right) - \left[\boldsymbol{g} \otimes, \boldsymbol{V}^{(0)} \right]_- + \boldsymbol{G} + \boldsymbol{D}. \quad (6)$$

4 Final reconstruction

The unknown remaining diagonal piece $\boldsymbol{D}$ can be reconstructed by forming the forward limit to splitting functions, e.g. ${}^{QQ}P(z) = \mathrm{LIM}\,{}^{QQ}V(x,y) \equiv \lim_{\xi\to 0} {}^{QQ}V(\frac{z}{\xi},\frac{1}{\xi})/|\xi|$. Comparing it with the known two-loop DGLAP kernels one represents $\boldsymbol{D}$ as a convolution of simple kernels whose non-forward counterparts are easy to find. Since $\mathrm{LIM}\,\{V_1 \otimes V_2\} = \mathrm{LIM}V_1 \otimes \mathrm{LIM}V_2$ restoration of D from the forward case is simple and one gets a complete $\boldsymbol{V}^{(1)}$[1].

References

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